Exploring the mechanism of active noice cancelation

Mathematical Exploration

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# Introduction

I have always found music inspiring, and I always carried around a Walkman with me where ever I go when I was young. I grew up in a crowded city, and loud noises in public, such as the subway station and the sidewalk, have always bothered me when I listen to music. This is why when my parents bought me my first active noise canceling (ANC) headphones when I was in middle school, I was convinced that it functioned on magic.

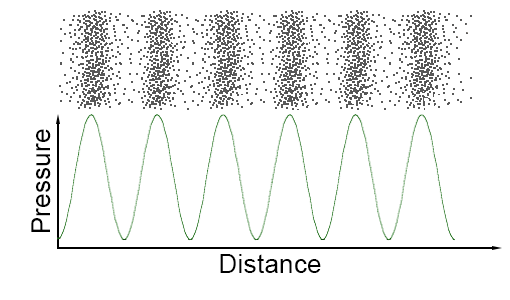
 I learned the principals which ANC function on last year in my SL physics course. Sound is a pressure wave of air with propagating numbers of air molecules as moving across the sound wave. There are points of compression created where there are more air molecules present with higher pressure, and points of rarefaction where there are fewer air molecules present with lower pressure.

Figure 1. Each dot on the top represent a air molecule, allowing the first graph to represent what a soundwave look like in real life. The second graph shows the air pressure at different points in the sound wave.

Figure one includes a graph that represents the shown sound wave in the form of a sine function. While most sound waves cannot be represented by a single sine function, this shows soundwave’s function like characteristic as each point in space only have one air pressure.

The graph in figure one denoted soundwaves by graphing distance against air pressure to better math with the visual representation of soundwave above, but this is not how soundwaves are usually recorded. Microphone record sound through the use of a diaphragm, which vibrates along the air molecule in the air as sound pass by. The microphone records the change in the physical location of the diaphragm, which is caused by the different air pressure on the soundwave passing by, allowing the computer to note down the change in air pressure over the change in time.

When two different soundwaves overlap, they interfere with each other. By viewing soundwave as a function of the change of air pressure over the change in time, it is intuitive that the result of two functions overlapping can be found by adding the two functions together. The graph shows the result of the interference, h(x), of the two functions f(x) and g(x). In the green boxes, because of that both soundwaves have the same sign, they interfere constructively, causing the amplitude of the resulting wave h(x) to be greater than either f(x) or g(x) in those regions. In boxes highlighted by red, the soundwaves interfere destructively, as their difference signs causes the two sound waves to cancel each other out. Consequentially, the resulting wave h(x) has a smaller amplitude than either f(x) or g(x) in these regions.

ANC headphones uses the interference of soundwaves to block out the noises from the outside environment. The headphone’s tight seal on the ears act as a physical barrier of sound, but there will still be some noise getting in to the headphone. The headphone then uses multiple different microphones to record the noise that was not blocked out by the headphone, and flips the signal to completely cancel it.

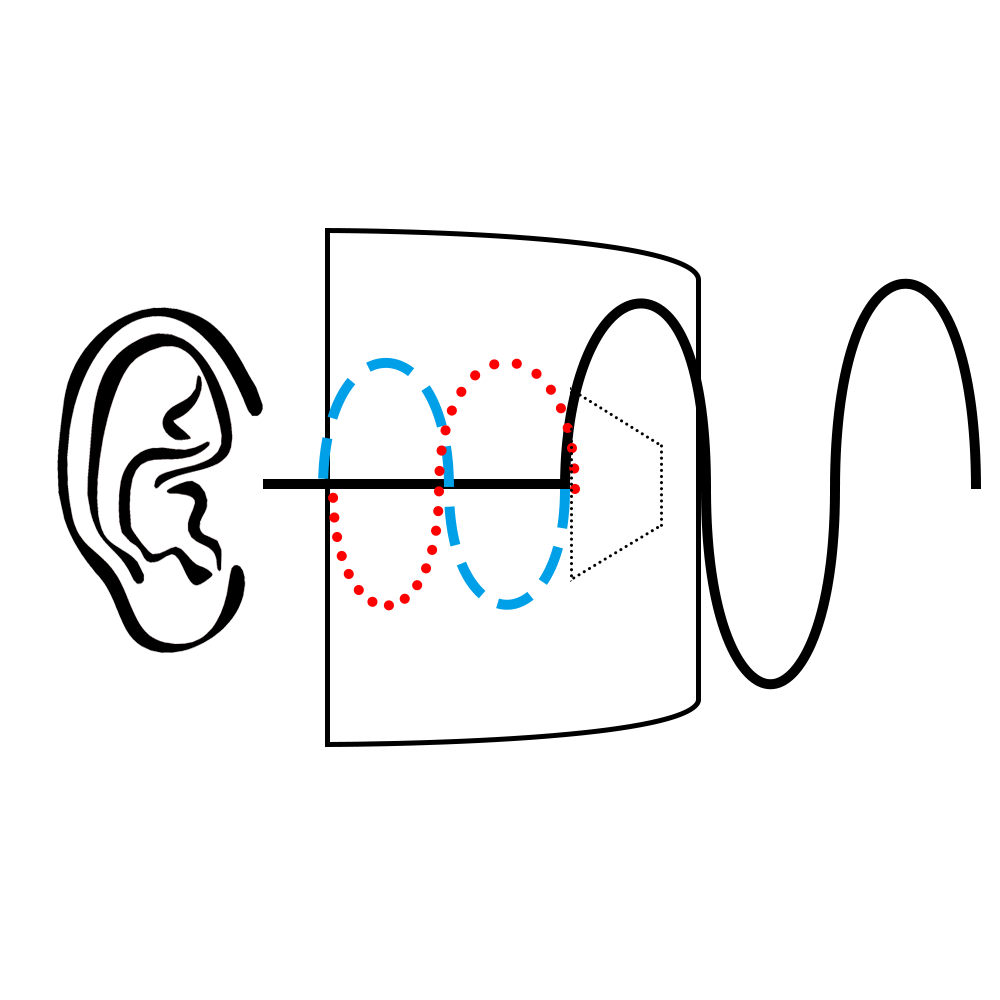


Figure 2 The black line represents the noise from outside. The blue line is the noise heard by the ears without ANC on, and the red line is the sound coming out from the speakers.

While some cheaper headphones use an analogue circuit to directly flip the signal from the microphone to cancel the sound, this type of noise cancelation does not perform as well as many would expect. Because of that high precision is required to cancel out the sound because of the high speed of sound, the uncertainties created by changes in temperature would affect the effect of the noise cancelling circuit.

Moreover, different frequencies of sound are blocked out by different amounts by the earphones themselves. Since there will always be a barrier between the microphone and the speakers, such as circuit boards even when the speaker is placed inside the earphones, the earphone needs to adjust the amplitude of different frequencies of sound.

# The Fourier Theorem

In order to be able to manipulate the amplitude of different frequencies of sound, the headphone must separate the different frequencies of sounds first. Jean-Baptiste Joseph Fourier, a French mathematician and physicist, have tackled this problem. His Fourier theorem states that any periodical function can be composed of a Fourier Series.

A Fourier Series is a summation function of an infinite amount of sine and cosine functions of different frequencies. Since the Fourier theorem is commonly regarded as an intuitive assumption in math, and its proof goes beyond the scope of this essay, I will be providing an example to show the theorem’s affect instead of going through a tedious proving process.

This graph illustrates the function . As n increases from 1 to infinity, the graph approaches the desired shape of the function which it is trying to construct. If a recording of sound was imagined as a self-repeating function, it could also be expressed as a Fourier Series. By expressing soundwaves in the form of a Fourier Series, to be able to adjust the amplitude of each frequency, the headphone will be better able to cancel out noises actively.

# Fourier Analysis

I have established that a soundwave can be represented using a Fourier Series, and now I will investigate the concept of the Fourier analysis, which allows the finding of the coefficients in the Fourier Series.

## Fourier Transformation

Starting simple, human’s understanding of what defines a sinusoidal wave needs to translated into mathematics. Take the function f(x) = sin(x) for example, the amplitude of the function is just f(x) for any given point on the x axis. We determine the period of the function by first finding an x-interception of the function. We then fallow the function, as it increases above the original point, return to the height of the original point, decreases to below the original point and finally return again to the height of the original point. We then find the distance the we have traveled in the x-axis, and that value will be the period of the function.

While we can replicate the identification process exactly with the use of f’(x) and f(x), which finds the slope of the function and x-interceptions, the process can be dramatically simplified with another way of approaching the problem.

The characteristic of a positive hump directly followed by a negative hump both with the exact same shape can be taken advantage of to describe a period. The opposite nature of the humps cancels each other out when trying to find the average or the integral of the graph f(x) within the boundaries of its period. When the boundaries do not match up exactly with the period, not all of the parts above and below the x-axis with match up perfectly. Causing the average and the integral of the graph f(x) to have an absolute value greater than 0.

Now, recall the formula for the Fourier series:

A key difference between G(x) and f(x), which we have used to investigate traits separating different sine functions ins that g(x) also includes functions of cosine. Cosine function are essentially sine functions that are phase-shifted. However, since there are no phase-shifts included in the series on the sine or cosine functions, the cosine functions cannot be simply turned into sine functions.

A nice tool that can help us create a function that takes in the consideration of both sine and cosine is the complex number plane: complex numbers can be expressed in the polar form , which creates a connection between the function between both sine and cosine. This being said, the Fourier series cannot be easily converted to a list of cis functions, as each sine and cosine wave of same frequency may have different coefficients and , making the cis function complex and difficult to simplify:

Since finding an explicit pattern to separate the frequencies from observing the Fourier is difficult, we can instead approach this problem from thinking how to obtain the weight of each frequency from a function that we would like to express using the Fourier series.

A key difference between g(x) and f(x) which we have used to investigate traits separating different sine functions is that g(x) is also partially made out of cosine functions. Without any phase shift existing in the Fourier transform, the cosine and sine functions with the same period will interfere with each other, as they both cancel each other out in the same frequency.